

# Problem Set

MA18Q1-O

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## OLG: Government purchases in the OLG model

Consider the following optimization problem.

$$\begin{aligned} \max_{c_t^Y, c_{t+1}^O, s_t} & \frac{(c_t^Y)^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{(c_{t+1}^O)^{1-\theta}}{1-\theta} \\ \text{subject to} & \\ & c_t^Y + s_t + G_t = w_t, \\ & c_{t+1}^O = (1+r_{t+1})s_t, \end{aligned}$$

where  $G_t$  is the lump-sum tax. Assume

$$\theta = 1, \quad \text{and} \quad F(K, AL) = K^\alpha (AL)^{1-\alpha}.$$

1. Under the assumption that  $A_{t+1} = (1+g)A_t$  and  $L_{t+1} = (1+n)L_t$ , derive the dynamic equation

$$\hat{k}_{t+1} = \frac{1}{(1+g)(1+n)(2+\rho)} \left[ (1-\alpha)\hat{k}_t^\alpha - \hat{G}_t \right],$$

where  $\hat{G} = G/A$  and  $\hat{k} := K/(AL)$ .

2. Suppose that the economy is on the balanced growth path (i.e.,  $\hat{k}_{t+1} = \hat{k}_t$ ) with  $\hat{G}_t = 0$ . At the beginning of period  $t = 0$ , the government puts a permanent tax increase in force, effective immediately. Describe what would happen after that.
3. Suppose that the economy is on the balanced growth path (i.e.,  $\hat{k}_{t+1} = \hat{k}_t$ ) with  $\hat{G}_t = 0$ . At the beginning of period  $t = 0$ , the government announces a permanent tax increase as of  $t = 2$ . Describe what would happen after this announcement.

## Modify OLG

In the textbook OLG model, consumers do not care about their offsprings' welfare, which might be unsatisfactory for you. Rewrite the model to modify this point. In particular, include the following considerations in the textbook OLG model.

- Each individual receives utility from leaving a legacy.
- The legacy is equally shared by their children, at the end of their youth.

Solve this problem and explain the difference from the textbook model, if any.

## Ricardian Equivalence

Consider the budget constraint for consumers

$$C(t) + \dot{S}(t) = r(t)S(t) + W(t) - T(t) \quad \text{with} \quad \lim_{t \rightarrow \infty} e^{-\int_0^t r(s)ds} S(t) \geq 0, \quad (1)$$

where  $C, S, r, W, T$  are consumption, savings, real interest rate, wage and tax, respectively, and that for the government

$$\dot{D}(t) = r(t)D(t) + G(t) - T(t) \quad \text{with} \quad \lim_{t \rightarrow \infty} e^{-\int_0^t r(s)ds} D(t) \leq 0, \quad (2)$$

where  $G(t)$  and  $D(t)$  are government purchases and government debt.

1. Use (1) to derive the lifetime budget constraint for the consumer.
2. Use (2) to derive the “lifetime” budget constraint for the government.
3. Show that taxes and debt financing have an equivalent effect on the consumer’s total utility  $\int_0^\infty e^{-\rho t} u(C(t)) dt$ . That is, the government’s plan about how and when to raise tax in order to finance the government purchases  $G(t)$  does not affect the utility.