Problem Set

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Discrete-time Ramsey model

To make the model simple, assume that there is no population growth ($n = 0$) or techinical growth ($g = 0$). Assume further that $\delta = 1$. We omit hat ($\hat{}$) for simplicity.

The capital accumulation equation, under Cobb–Douglas production function, is

$$k_t = k_{t-1}^{\alpha} - c_t, \quad 0 < \alpha < 1$$

where k_t is the capital per capita for period t, c_t consumption per capita. The representative consumer solves

$$\max \sum_{t=1}^{\infty} \beta^{t-1} \ln (k_{t-1}^{\alpha} - k_t) =: V(k_0)$$

where $k_0 > 0$ is given and β ($0 < \beta < 1$) is the discount factor. The Bellman equation is expressed as

$$V(x) = \max_{y} [\ln (x^{\alpha} - y) + \beta V (y)]$$

Our goal here is to find a function for which the optimal trajectory $(k_t)_{t \geq 0}$ satisfies $k_t = h(k_{t-1})$ for all $t \geq 1$.

1. Suppose that the value function has the form $V(x) = a + b \ln x$. Derive the maximizer y as a function of x; $y = h(x)$ is a candidate for the optimal policy $k_t = h(k_{t-1})$.

2. Find a and b such that $V(x) = \ln (x^{\alpha} - h(x)) + \beta V(y)$ holds true.

3. Find optimal policy function h such that $k_t = h(k_{t-1})$ for optimal path $k_0, k_1, k_2, \ldots$ and draw a staircase diagram to show that the optimal capital accumulation path converges to a finite limit k^*, from which you can be sure that you made a correct guess.