Problem Set

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[1] CRRA utility function

CRRA (Constant Relative Risk Aversion) functions are an often-used class of utility functions in macroeconomics. Typically, CRRA functions have the following form:

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta} & \text{if } \theta \neq 1 \text{ and } \theta \geq 0, \\ \ln c & \text{if } \theta\text{=1.} \end{cases}$$

1. Show that the CRRA functions have constant relative risk aversion; that is constant $-\frac{cu''(c)}{u'(c)}$. More specifically,
$$-\frac{cu''(c)}{u'(c)} = \theta, \qquad \text{for all } c > 0.$$

2. Show that
$$\lim_{\theta \to 1} \frac{c^{1-\theta} - 1}{1 - \theta} = \ln c.$$
[Hint: For $a > 0$, $\frac{d}{dx}(a^x) = a^x \ln a$. Use l'Hôpital's theorem.]

[2] Cake eating problem

You have $w(0)$ kilogram of cake at time $t = 0$. The amount of cake at t, $w(t)$, follows the differential equation
$$\dot{w}(t) = -c(t),$$
where $c(t)$ [kg/min] is the instantaneous speed of consumption at time t. Find a consumption stream $c(t)$ that maximizes your utility,
$$U = \int_0^\infty e^{-\rho t} \ln c(t) dt,$$
where $\rho > 0$ is a constant discount rate.

3. Set up the Hamiltonian for the problem.

4. Derive the differential equation that c obeys.

5. Use $w(0) = \int_0^\infty c(t)dt$, which states that you are going to eat up the whole cake, to fully determine $c(t)$. [Optimality requires a condition similar to this one.]