Problem Set

MA18Q1-J

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[1] Capital accumulation equation

The capital accumulation equation for the Ramsey model is given by

$$\dot{\hat{k}} = f\left(\hat{k}\right) - \hat{c} - (\delta + g + n)\hat{k},$$

where $\hat{k}$ is the capital stock per unit of effective labor, $\hat{c}$ the consumption per unit of effective labor. Parameters δ, g, n are the depreciation rate, technical growth rate, population growth rate, respectively.

1. Suppose that the intensive-form production function is of Cobb–Douglas form, $f\left(\hat{k}\right) = \hat{k}^{\alpha}$. Draw a locus of $\left(\hat{k}, \hat{c}\right)$ on which $\dot{\hat{k}} = 0$ is met.

2. Indicate the golden rule level of capital stock $\hat{k}^*_G$ in the same graph.

[2] Utility

Consider the total utility function

$$\int_0^{\infty} e^{-\rho t} u\left(\hat{c}(t)\right) dt$$

1. Compute the total utility for a steady state level of consumption $\hat{c}(t) \equiv \bar{c}$; i.e. $\int_0^{\infty} e^{-\rho t} u\left(\bar{c}\right) dt$. [Hint: Use the integration by parts formula.]

2. Compute the marginal rate of substitution between $\hat{c}(t)$ and $\hat{c}(s)$ for $t < s$.

3. Explain why you can interpret ρ as a measure of impatience. Use the above results.

Integration by parts formula

$$\int_a^b f(x)g'(x)dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x)dx,$$

where $[f(x)g(x)]_a^b = f(b)g(b) - f(a)g(a)$.

Marginal rate of substitution Let c_i be consumption of good $i = 1, 2, \ldots, N$ and $U(c_1, \ldots, c_N)$ a utility function. The marginal rate of substitution between goods i and j is defined by

$$\frac{\partial U / \partial c_i}{\partial U / \partial c_j}.$$


$\hat{c}$
O
$\hat{k}$