Problem Set

MA18Q1-I

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Before starting the model construction, let's see how the Ramsey model behaves. The now familiar phase diagram analysis will work perfectly well.

Diagram for the Ramsey Model

With some parameter specifications ($g = n = 0$, $\theta = 1$), the dynamics of the endogenous variables (k, c), per-capita capital and consumption, of the Ramsey model is governed by the following system of differential equations. $\rho > 0$ is a parameter for impatience.

$$\dot{k} = f(k) - c - \delta k$$
$$\frac{\dot{c}}{c} = f'(k) - \delta - \rho$$

1. Complete Figure 1: Draw curves that correspond to

$$\dot{k} = 0 \quad \text{and} \quad \dot{c} = 0.$$

2. You see four regions didived by the loci. Determine the signs of $\dot{k}$ and $\dot{c}$ in each region.

The above system of differential equations determines the trajectory from a given $(k(0), c(0))$. In the Ramsey model, however, $k(0)$ is given but $c(0)$ is not. So, there are infinite possibilities regarding the choice of initial $(k(0), c(0))$, which lies somewhere on the dotted vertical line in Figure 2. To understand how to choose the right initial value, you must understand how the trajectory from an arbitrary initial value looks like.

3. Complete Figure 2: Draw the loci, $\dot{k} = 0$ and $\dot{c} = 0$ again and then draw a sketch of trajectories departing from the four candidates for the "initial" position depicted as black dots.

4. As a matter of fact, the trajectory starting from the optimal initial position converges to the steady state. Find this optimal trajectory.

(1) and (2)

Figure 1: (k, c) plane

(3) and (4)

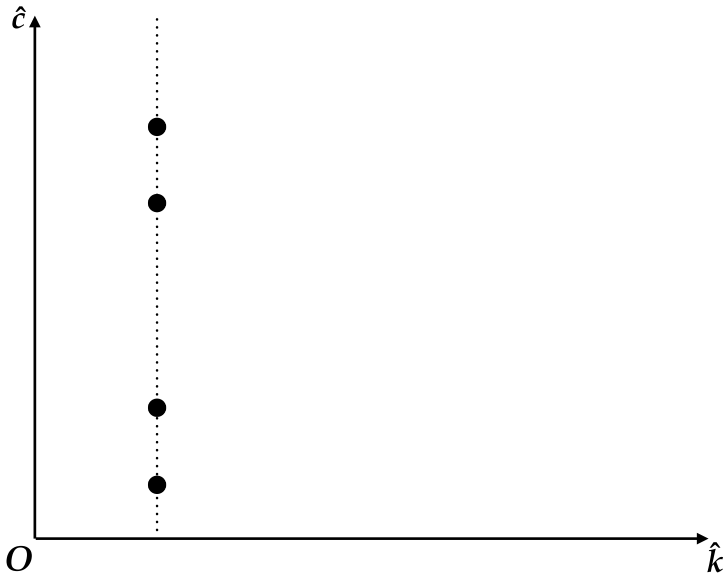

Figure 2: Draw trajectories from the dots