Problem Set

MA18Q1-G

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[1] Mankiw–Romer–Weil Model

Derivation of the differential equations Let $0 < \alpha < 1$ and $0 < \beta < 1$ with $\alpha + \beta < 1$. The output is given by

$$Y = K^\alpha H^\beta (AL)^{1-\alpha-\beta}.$$

Capital accumulation equations are given by

$$\dot{K} = s_k Y - \delta K, \qquad \dot{H} = s_h Y - \delta H,$$

where they assume K and H have the same depreciation rate, δ. Define $y = Y/AL$, $k = K/AL$, $h = H/AL$ and show that the following two-dimensional differential equation system determines the dynamics of the model:

$$\begin{aligned}\dot{k} &= s_k k^\alpha h^\beta - (\delta + g + n)k \\ \dot{h} &= s_h k^\alpha h^\beta - (\delta + g + n)h.\end{aligned}$$

Convergence to the steady state

1. Show that

$$\dot{k} = 0 \Leftrightarrow k = \left(\frac{s_k}{\delta + g + n}\right)^{\frac{1}{1-\alpha}} h^{\frac{\beta}{1-\alpha}}, \tag{1}$$

$$\dot{h} = 0 \Leftrightarrow h = \left(\frac{s_h}{\delta + g + n}\right)^{\frac{1}{1-\beta}} k^{\frac{\alpha}{1-\beta}}. \tag{2}$$

2. Similarly, derive conditions for $\dot{k} > 0$, $\dot{k} < 0$, $\dot{h} > 0$ and $\dot{h} < 0$.

3. Equations (1) and (2) divide (k, h) space into four regions. See Figure 1 on the answer sheet. For each region, determine the sign of $\dot{k}$ and $\dot{h}$, and circle the correct inequality in Figure 1.

4. Now you can draw a sketch of dynamic behavior of the two-dimensional system. Draw trajectories starting from each of the eight dots in Figure 2.

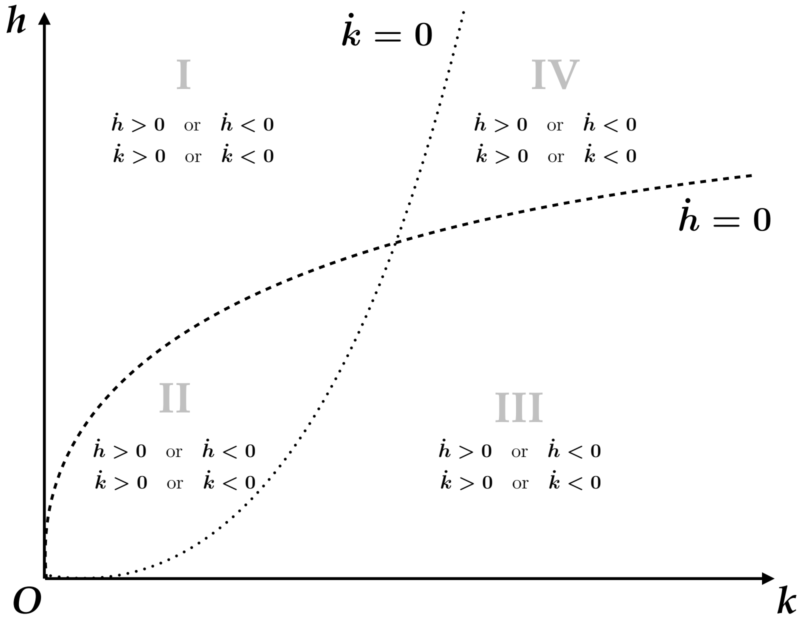

Figure 1: Determine the signs of $\dot{k}$ and $\dot{h}$

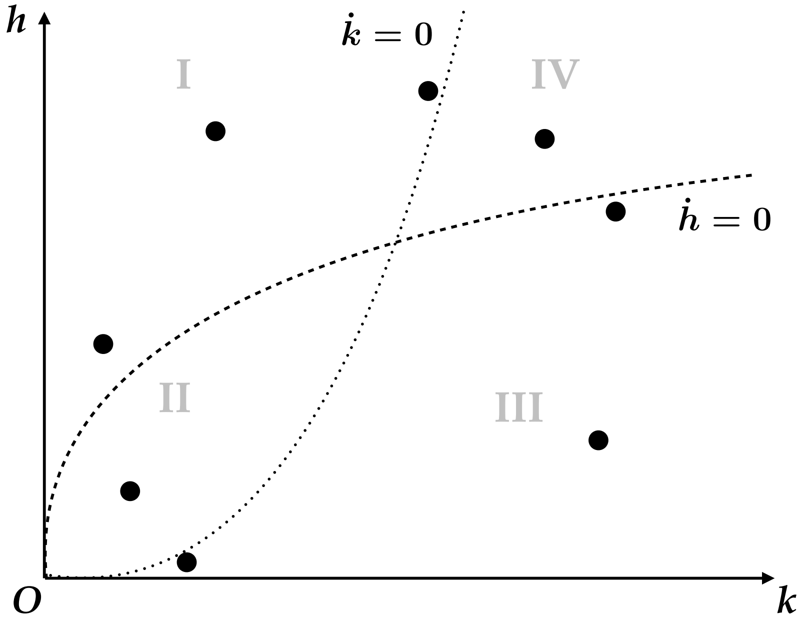

Figure 2: Draw trajectories from the dots