Problem Set

MA17Q4-M

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2018/1/29

Pay-as-you-go social secutiry [Romer, Problem 2.17a]

Consider a Diamond economy where $g = 0$, $f(k) = k^\alpha$, $\theta = 0$. Suppose that the government taxes each young individual an amount T and pays benefits, B, to old individuals. Each individual solves the following maximization problem:

$$\max_{c_t^Y, c_{t+1}^O, s_t} \ln c_t^Y + \frac{\ln c_{t+1}^O}{1+\rho}$$

subject to

$$c_t^Y + s_t + T = w_t,$$
$$c_{t+1}^O = (1 + r_{t+1})s_t + B,$$

1. Derive the saving function $s_t = s\left(r_{t+1}, w_t, T, B\right)$.
2. Derive the dynamic system $k_t \mapsto k_{t+1}$ using capital market clearing condition, $K_{t+1} = s_t L_t$, and the condition for government's balanced budget, $B = (1+n)T$.
3. Compared to the simplest case with $T = B = 0$, how does this pay-as-you-go social security affect the balanced-growth-path value of k?
4. If the economy is initially on a balanced growth path that is dynamically efficient, how does a marginal increase in T affect the welfare of current and future generations?
5. What happens if the initial balanced growth path is dynamically inefficient?