# Problem Set

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## Discrete-time Ramsey model

To make the model simple, assume that there is no population growth ($n = 0$) or techinical growth ($g = 0$). Assume further that $\delta = 1$. We omit hat ($\hat{}$) for simplicity.

The capital accumulation equation, under Cobb–Douglas production function, is

$$k_t = k_{t-1}^{\alpha} - c_t, \quad 0 < \alpha < 1$$

where $k_t$ is the capital per capita for period $t$, $c_t$ consumption per capita. The representative consumer solves

$$\max \sum_{t=1}^{\infty} \beta^{t-1} \ln\left(k_{t-1}^{\alpha} - k_t\right) =: V(k_0)$$

where $k_0 > 0$ is given and $\beta$ $(0 < \beta < 1)$ is the discount factor. The Bellman equation is expressed as

$$V(x) = \max_{y} \left[\ln\left(x^{\alpha} - y\right) + \beta V\left(y\right)\right]$$

1. Suppose that the value function has the form $V(x) = a + b \ln x$. Derive the candidate for $y = h(x)$.
2. Find $a$ and $b$ such that $V(x) = \ln\left(x^{\alpha} - h(x)\right) + \beta V(y)$ holds true.
3. Find optimal policy function $h$ such that $k_t = h(k_{t-1})$ for optimal path $k_0, k_1, k_2, \ldots$ and draw a staircase diagram to show that the optimal capital accumulation path converges to a finite limit $k^*$, from which you can be sure that you made a correct guess.