Problem Set

MA17Q4-J

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Diagram for the Ramsey Model

The dynamics of the endogenous variables $(\hat{k}, \hat{c})$ of the Ramsey model is governed by the following system of differential equations.

$$\dot{\hat{k}} = f\left(\hat{k}\right) - \hat{c} - (\delta + g + n)\hat{k}$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{f'\left(\hat{k}\right) - \delta - \rho - \theta g}{\theta}$$

1. Complete Figure 1: Draw curves that correspond to

$$\dot{\hat{k}} = 0 \quad \text{and} \quad \dot{\hat{c}}\text{=0.}$$

2. You see four regions didived by the loci. Determine the signs of $\dot{\hat{k}}$ and $\dot{\hat{c}}$ in each region. (Do something similar to what you did for the MRW model.)

The above system of differential equations determines the trajectory from a given $(\hat{k}(0), \hat{c}(0))$. In the Ramsey model, however, $\hat{k}(0)$ is given but $\hat{c}(0)$ is not. So, there are infinite possibilities regarding the choice of initial $(\hat{k}(0), \hat{c}(0))$, which lies somewhere on the dotted vertical line in Figure 2. To understand how to choose the right initial value, you must understand how the trajectory from an arbitrary initial value looks like.

3. Complete Figure 2: Draw the loci, $\dot{\hat{k}} = 0$ and $\dot{\hat{c}} = 0$ again and then draw a sketch of trajectories departing from the four "initial" position depicted as black dots.

4. The trajectory starting from the optimal initial position converges to the steady state. Draw the trajectory.

(1) and (2)

Figure 1: $(\hat{k}, \hat{c})$ plane

(3) and (4)

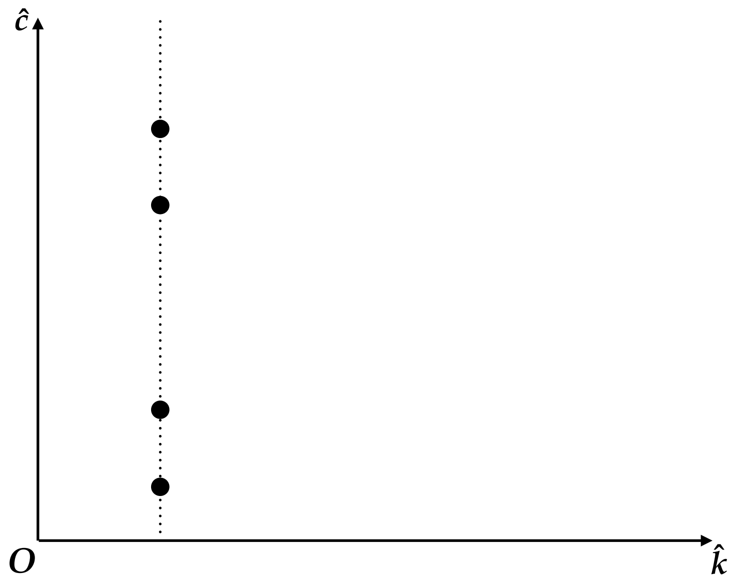

Figure 2: Draw trajectories from the dots