

Problem Set

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Government purchases in the OLG model

Consider the following optimization problem discussed in the class.

$$\max_{c_t^Y, c_{t+1}^O, s_t} \frac{(c_t^Y)^{1-\theta}}{1-\theta} + \frac{1}{1+\rho} \frac{(c_{t+1}^O)^{1-\theta}}{1-\theta}$$

subject to

$$c_t^Y + s_t + G_t = w_t,$$

$$c_{t+1}^O = (1 + r_{t+1})s_t.$$

Assume

$$\theta = 1, \quad \text{and} \quad F(K, AL) = K^\alpha (AL)^{1-\alpha}.$$

- (1) Explain all the variables and parameters. Interpret each term in the optimization problem.
- (2) Under the assumption that $A_{t+1} = (1 + g)A_t$ and $L_{t+1} = (1 + n)L_t$, derive the dynamic equation

$$\hat{k}_{t+1} = \frac{1}{(1+g)(1+n)(2+\rho)} \left[(1-\alpha)\hat{k}_t^\alpha - \hat{G}_t \right],$$

where $\hat{G} = G/A$ and $\hat{k} := K/(AL)$.

- (3) Suppose that the economy is on the balanced growth path with $\hat{G}_t = 0$. At the beginning of period $t = 0$, the government announces a permanent tax increase as of $t = 2$. Describe what would happen after this announcement. 1. Be sure to explain both long-run and short-run effects. In particular, describe the transition path to the final state in detail.

Answer sheet. Please write your name and id number.