Problem Set

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Alternative route to the FoC

Recall the consumer's optimization problem.

$$
\begin{aligned}
&\max_{c_t^Y, c_{t+1}^O, s_t} \frac{\left(c_t^Y\right)^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{\left(c_{t+1}^O\right)^{1-\theta}}{1-\theta} \\
&\text{subject to} \\
&\quad c_t^Y + s_t = w_t \\
&\quad c_{t+1}^O = (1 + r_{t+1})s_t.
\end{aligned}
$$

(1) Eliminate s_t from the budget constraints to obtain the life-time budget constraint for the young.

(2) Obtain the first-order condition by utilizing the proposition that optimality equates the marginal rate of substitution between c_t^Y and c_{t+1}^O and the slope of the budget line derived in (1).

Cobb–Douglas production and logarithmic utility

Recall the dynamic equation

$$
\hat{k}_{t+1} = \frac{s\left(f'(\hat{k}_{t+1})\right)\left[f(\hat{k}_t) - \hat{k}_t f'(\hat{k}_t)\right]}{(1+g)(1+n)}.
$$

Under the assumptions that $f(\hat{k}) = \hat{k}^\alpha$ and $u(c) = \ln(c)$, we can rewrite this as

$$
\hat{k}_{t+1} = \frac{(1-\alpha)}{(1+g)(1+n)(2+\rho)}\hat{k}_t^\alpha.
$$

Prove this fact.

Answer sheet. Please write your name and id number.