Macroeconomics 17q2
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Problem Set

mail@**kenjisato**.jp

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[1] Compounding with time-variable interest rate

Let $x(t)$ denote the bank balance at time t, where the initial balance is $x(0)$. The annual nominal interest rate is time dependent, which is denoted by $r(t)$. If there is not further saving or dissaving, the bank balance at time t can be computed by solving the following differential equation.

$$\dot{x}(t) = r(t)x(t).$$

(a) Show that the solution is given by

$$x(t) = x(0)e^{\int_0^t r(s)ds}.$$

(b) Explain why the discount factor of this economy should be $e^{-\int_0^t r(s)ds}$. In theory, the fair valuation of a project that will make a sure profit of 1 million dollars in t years is $e^{-\int_0^t r(s)ds}$ million dolloars.

[2] CRRA utility function

As a choice of utility function, CRRA (Constant Relative Risk Aversion) functions are often used in macroeconomics. The following functions are instances of CRRA functions:

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta} & \text{if } \theta \neq 1 \text{ and } \theta \geq 0, \\ \ln c & \text{if } \theta\text{=1.} \end{cases}$$

(a) Show that the CRRA functions have constant relative risk aversion; that is constant $-\frac{cu''(c)}{u'(c)}$. More specifically,

$$-\frac{cu''(c)}{u'(c)} = \theta, \qquad \text{for all } c > 0.$$

(b) Show that

$$\lim_{\theta \to 1} \frac{c^{1-\theta} - 1}{1-\theta} = \ln c.$$

Answer sheet. Please write your name and id number.