A background image of a black hole with a bright, glowing accretion disk on the right side. A small, dark sphere is visible in the center of the frame, set against the dark space of the black hole's event horizon.

How space travel is revolutionized with this one weird trick from chaos theory

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thanking Sven Prüfer and Matthias Hutzler

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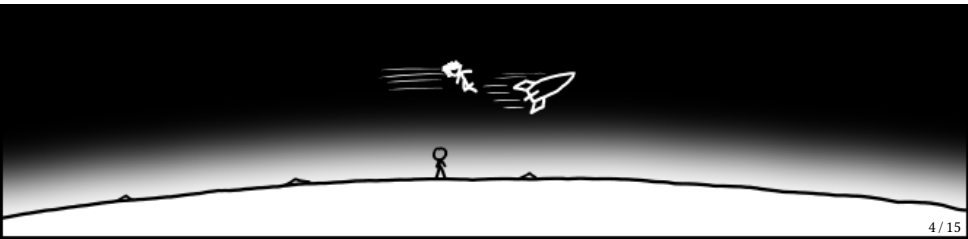


- 1 A crash course on orbital mechanics
 - Basic facts
 - Changing orbits
 - The tyranny of the rocket equation

- 2 One weird trick from chaos theory
 - Lagrangian points
 - Weak stability boundaries
 - The rescue of the Hiten
 - In nature

Part I

A crash course on orbital mechanics



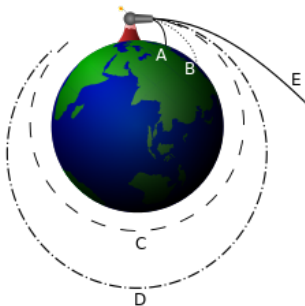
Basic facts

- Getting to space is easy.
The hard part is staying there.
- Gravitational acceleration at the height of the ISS is still $\approx 8.7 \text{ m/s}^2$.



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The hard part is staying there.



$$F_{\text{centripetal}} = F_{\text{gravitation}} \leadsto v_1 = \sqrt{GM/r}$$
$$E_{\text{kinetic}} = E_{\text{gravitation}} \leadsto v_2 = \sqrt{2} v_1$$

Basic facts

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The hard part is staying there.

body	second escape velocity
Earth	$11.2 \text{ km/s} \approx 40\,000 \text{ km/h}$
Moon	2.4 km/s
Sun	618 km/s
Milky Way	$\approx 550 \text{ km/s}$

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Basic facts

- Getting to space is easy.
The hard part is staying there.
- Velocity is very important.
- In the **one-body problem**, there are only three kinds of orbits: elliptic, parabolic, and hyperbolic.
- Have your models straight: Earth is ...
 - 1 a perfect ball?
 - 2 has atmosphere?
 - 3 rotating?

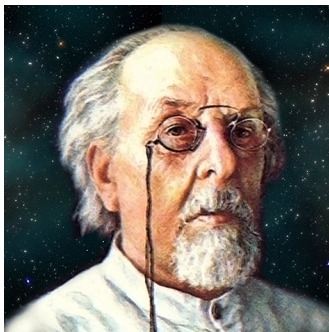
Changing orbits

“Live demo”

- Changing the phase
- Changing the eccentricity
- Changing the radius
- Changing inclination



The tyranny of the rocket equation



Konstantin Tsiolkovsky (* 1857, † 1935)

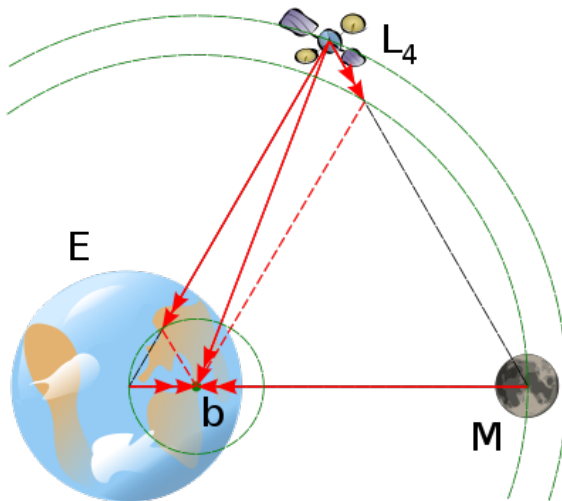
$$m_{\text{total}} = m_{\text{payload}} \cdot e^{\Delta v / v_{\text{eff. exhaust}}}$$

Part II

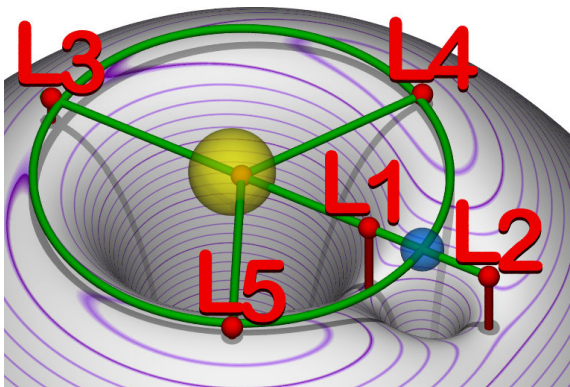
One weird trick from chaos theory



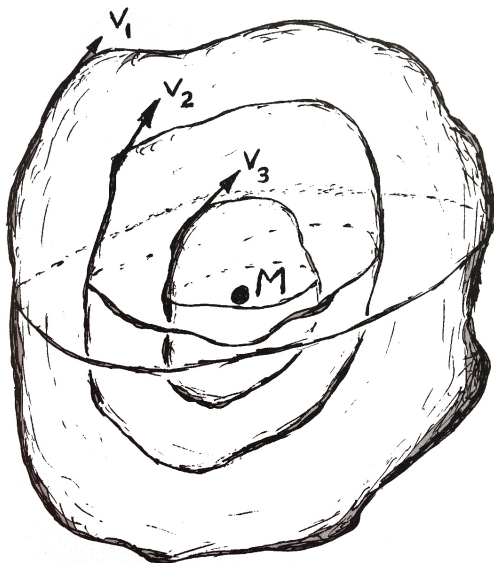
Lagrangian points



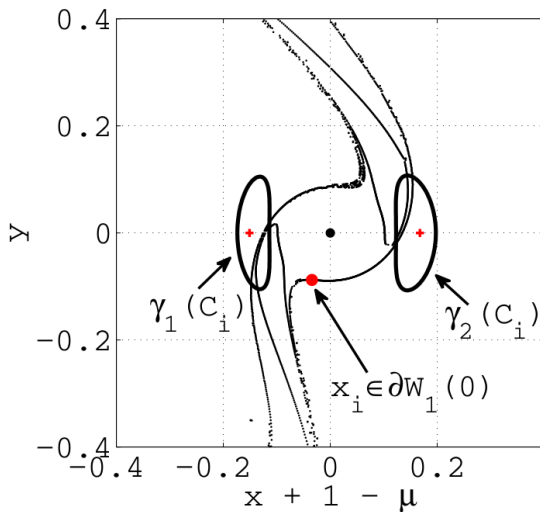
Lagrangian points



Weak stability boundaries

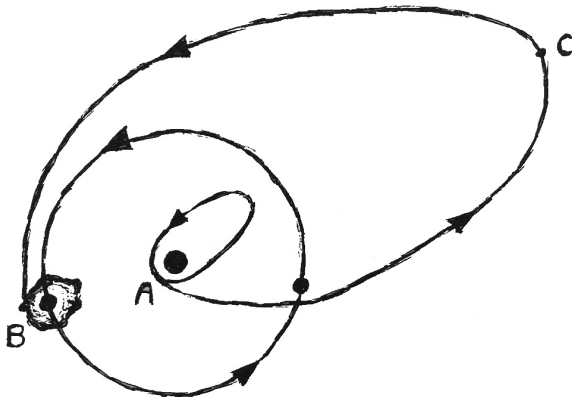


Weak stability boundaries





The rescue of the Hiten



In nature



Figure 10. Stars stream outward from the Tadpole Galaxy (Arp 188) along a tubelike channel that stretches for some 280,000 light-years. This conduit (the galactic equivalent of the tubes making up the interplanetary transport network) arose through gravitational interaction with a compact galaxy that can now be seen lurking behind one of the Tadpole's spiral arms. (Courtesy of ACS Science & Engineering Team and NASA.)