

Introduction to Mathematics for Political Science

Problem Set 9: Matrix Inversion and Determinants

Instructions: You are encouraged to work in groups and actively participate on the Piazza page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. Submit typed solutions using the link on the course page.

1. Consider the following system of equations:

$$\begin{aligned}3x_1 - x_2 &= 10 \\ -x_1 + 4x_2 &= 4\end{aligned}$$

Write this system in $A\mathbf{x} = \mathbf{b}$ form and solve via matrix inversion.

2. Let $C = AB$ where C is invertible and A and B are square matrices. Solve for A^{-1} .¹
3. Let $M = ABC$ where M is invertible and A , B , and C are square matrices. Solve for B^{-1} .²
4. If B is the inverse of A^2 , show that AB is the inverse of A .³

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5. Let

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Multiply the two matrices. What are X^{-1} and Y^{-1} , assuming $ad \neq bc$.⁴

6. A is an *idempotent* matrix if and only if $AA = A$. Show that if A is symmetric ($A^\top = A$) and idempotent then $(I - A) = (I - A)(I - A)^\top$, where I is the identity matrix.

¹Strang p. 90 #12

²Strang p. 90 #13

³Strang p. 90 #18

⁴Strang p. 90 #16

7. Let $R_{m \times n}$ be a rectangular matrix ($m \neq n$) and $A_{m \times m}$ be a symmetric matrix. Show $R^T A R$ is also symmetric. What are the dimensions of this matrix?⁵
8. Show every orthogonal matrix A has determinant 1 or -1. Hint: Apply the product rule ($|AB| = |A||B|$) and the transpose rule ($|A| = |A^T|$) for determinants.⁶
9. Let $\mathbf{x} = \{x_1, \dots, x_{50}\}$ denote the number of electoral votes for each state. Let $\mathbf{y}_i = \{y_{i1}, \dots, y_{ij}, \dots, y_{i50}\}$ denote whether or not candidate $i \in \{R, D\}$ won the votes of a given state, where $y_{ij} \in \{0, 1\}$. Write an expression for the total number of votes for each candidate.

⁵Strang 117 #19

⁶Strang p. 252 #8