

# An Introduction to Mathematics For Political Science

## Problem Set 4

You are encouraged to work in groups and actively participate on the Piazza page. Submitted solutions must be your individual work. Do not use a calculator or search for solutions. Show all of your work. All solutions must be written in LaTeX.

### Optimization

1. Find all extrema (local and global) of the following functions on the specified domains, and state whether each extremum is a minimum or maximum and whether each is only local or global on that domain. In your answer, report both the max/min and argmax/armin.

a)  $f(x) = x^2 - 4x + 2$  on  $[0, 3]$

b)  $f(x) = 2x^3 - x$  on  $(-1, 1]$

c)  $f(x) = \sqrt{x}$  on  $[0, 4)$

d)  $f(x) = -x^2 + 4$  on  $(-2, 2)$

2. Explain (in words) the difference between a global maximum and a supremum.

3. Find the second derivative with respect to  $x$  of the following functions:

a)  $2x^3 - 4x^2 + x$

b)  $x^4 + e^{2x}$

c)  $\ln^2(x)$

d)  $-(x - a)^2$

e)  $xe^{-x}$

4. Find all critical points and inflection points of the following functions. Identify whether each critical point is a local maximum, local minimum, or inflection point.

a)  $f(x) = x^3 - 3x^2$

b)  $f(x) = x^3 - 6x^2 + 9x + 15$

c)  $f(x) = -(x - b)^2$

d)  $f(x) = -x^3$

5. Identify the regions of  $\mathbb{R}$  on which the following functions are weakly concave and/or convex:

a)  $f(x) = -x^3$

b)  $f(x) = \frac{1}{x}$

c)  $f(x) = x^3 - 3x^2$

d)  $f(x) = 4x - 5$