Non-Parametric Methods

What Are Nonparametric Methods?

- Common characteristics:
  - Independence of observations
  - Few assumptions
  - Dependent variable may be categorical
  - Focus on rank ordering/frequencies
  - Hypotheses: ranks, medians or frequencies
  - Sample sizes less stringent

Parametric or Nonparametric?

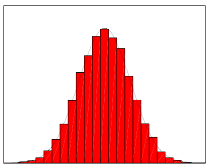
- Important to consider the type of data
- Nominal: non-parametric
- Ordinal, interval or ratio: less clear
- Size of sample
- Assumptions of the test
- Shape of distribution
- The choice is not simple
- Often parametric methods incorrectly chosen

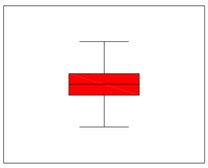
Situations Suggesting Nonparametric Methods

- Nominal independent/dependent variable
- Ordered data with many ties
- Rank-ordered data
- Small sample size/unequal groups
- Dependent variable shape not consistent with normal
- Groups: unequal variances
- Notable outliers

Are The Data Normally Distributed?

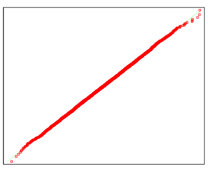
- Normality main assumption parametric tests
- Assessed by histograms, boxplots or qq plots
- Remember separate plots for separate groups



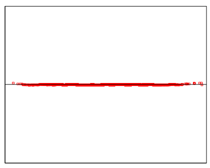


Are The Data Normally Distributed?

Normal probability plot



Detrended probability plot



When To Use Which Test

	RESPONSE		
NO OF SAMPLES	NOMINAL	ORDINAL OR NON-NORMAL	NORMALLY DISTRIBUTED
ONE SAMPLE	χ^2-test, Z-test	Kolmogorov-Smirnov Sign test	t-test
TWO SAMPLE – INDEPENDENT	χ^2-test (r x c), Fisher's exact test	Mann-Whitney U Median test	Unpaired t-test
TWO SAMPLE – PAIRED	McNemar's test Stuart-Maxwell test	Wilcoxon signed rank Sign test	Paired t-test
MULTIPLE SAMPLES (K>2) – INDEPENDENT	χ^2-test (r x k) Fisher-Freeman-Halton	Kruskal-Wallis test Median Test Jonckheere-Terpstra test	Analysis of variance (ANOVA)
MULTIPLE SAMPLES (K>2) – PAIRED	Cochran Q test	Friedman test Page test Quade test	Repeated measures ANOVA
ASSOCIATION BETWEEN TWO VARIABLES	Contingency coefficient Phi, r_ϕ Cramér, C	Spearman's rank Kendall's tau	Pearson product moment correlation
AGREEMENT BETWEEN TWO VARIABLES	Simple kappa	Weighted kappa	Limits of agreement

Median Test

- Uses chi-square statistic
- Useful when assumptions of Mann-Whitney U violated
- Assumptions:
  - At least ordinal level response
  - In 2 or more independent groups
  - Assumptions of chi-square test
- Hypotheses:
  - H_0: samples from populations same median
  - H_A: samples from populations differing medians

Method

- Decide on hypotheses and α
- Find overall median
- In each group classify as above/below median
- Arrange table groups by above/below median
- Carry out chi-square test on this table
- Compare with critical values of χ^2 distribution

Example

- Data from Sanjana Nyatsanza, Fulbourn hospital
- Three groups, dementia, differ on MMSE score?

Group	MMSE Score										
1	19	7	17	28	21	6	21	19	27	8	25
2	16	22	30	24	22	23	22	28	29	29	0
3	4	9	30	29	25	22	25	26	27	18	10

- Overall median = 22

Example

- Data from Sanjana Nyatsanza, Fulbourn hospital
- Three groups, dementia, differ on MMSE score?

Group	MMSE Score										
1	19	7	17	28	21	6	21	19	27	8	25
	−	−	−	+	−	−	−	−	+	−	+
2	16	22	30	24	22	23	22	28	29	29	0
	−	=	+	+	=	+	=	+	+	+	−
3	4	9	30	29	25	22	25	26	27	18	10
	−	−	+	+	+	=	+	+	+	−	−

- Overall median = 22

Group number	≤ 22	>22	Total
1	8	3	11
2	5	6	11
3	5	6	11
Total	18	15	33

6	5
6	5
6	5

- X^2=2.2 < 5.99 = χ^2_2
- Do not reject null hypothesis medians same for all groups
- No post hoc testing

- Presentation of results
  - The results of the median test show that there is no evidence of a difference in median MMSE scores between the three dementia groups (χ^2_2=2.2, p=0.33).
- Advantages and limitations:
  - Straightforward to apply
  - Useful when exact values unknown
  - Size of difference not accounted for
  - Less powerful than Mann-Whitney U and Kruskal-Wallis

When To Use Which Test

		RESPONSE		
NO OF SAMPLES		NOMINAL	ORDINAL OR NON-NORMAL	NORMALLY DISTRIBUTED
ONE SAMPLE		χ^2-test, Z-test	Kolmogorov-Smirnov Sign test	t-test
TWO SAMPLE	INDEPENDENT	χ^2-test (r x c), Fisher's exact test	Mann-Whitney U Median test	Unpaired t-test
	PAIRED	McNemar's test Stuart-Maxwell test	Wilcoxon signed rank Sign test	Paired t-test
MULTIPLE SAMPLES (K>2)	INDEPENDENT	χ^2-test (r x k) Fisher-Freeman-Halton	Kruskal-Wallis test Median Test Jonckheere-Terpstra test	Analysis of variance (ANOVA)
	PAIRED	Cochran Q test	Friedman test Page test Quade test	Repeated measures ANOVA
ASSOCIATION BETWEEN TWO VARIABLES		Contingency coefficient Phi, r_ϕ Cramér, C	Spearman's rank Kendall's tau	Pearson product moment correlation
AGREEMENT BETWEEN TWO VARIABLES		Simple kappa	Weighted kappa	Limits of agreement

Kruskal-Wallis Test

- Used for multiple independent samples
  - i.e. more than 2 groups
- Non-parametric equivalent of one-way Analysis of Variance (ANOVA)
- Extension of Mann-Whitney U test
  - Same results for 2 groups

Kruskal-Wallis Test

- Assumptions:
  - Two or more independent groups
  - At least ordinal dependent variable
  - Randomly selected observations
  - Population distributions same shape
- Hypotheses:
  - H_0: populations have the same median
  - H_0: populations have the same shape and spread

Method

- Construct hypotheses and decide on α
- Rank whole sample from smallest to largest
- Calculate sum of ranks for each group
- Calculate average rank for each group & overall
- Calculate:

$$H = \frac{12\sum n_i\left(\bar{R}_i - \bar{R}\right)^2}{N\left(N+1\right)}$$

- Compare to chi-square distribution with k–1 df

Example

- Altman's book, headache activity, 3 treatment groups

Relaxation and response feedback	Relaxation alone	Untreated
62	69	50
74	43	–120
86	100	100
74	94	–288
91	100	4
37	98	–76

Example

- Altman's book, headache activity, 3 treatment groups

Relaxation and response feedback	Rank	Relaxation alone	Rank	Untreated	Rank
62	8	69	9	50	7
74	10.5	43	6	–120	2
86	12	100	17	100	17
74	10.5	94	14	–288	1
91	13	100	17	4	4
37	5	98	15	–76	3
Rank sum	59		78		34
(average)	(9.83)		(13)		(5.67)

- Average rank for whole sample is 9.5
- Calculate the test statistic, H:

$$H = \frac{12\sum n_i\left(\bar{R}_i - \bar{R}\right)^2}{N\left(N+1\right)}$$

$$= \frac{12\times\left(6\times\left(9.83-9.5\right)^2+6\times\left(13-9.5\right)^2+6\times\left(5.67-9.5\right)^2\right)}{18\times\left(18+1\right)}$$

$$= 5.69$$

- $5.69 < 5.99 = \chi^2$ on $2df$
- Therefore insufficient evidence to reject H_0

- Presentation of the results:
  - The Kruskal–Wallis test indicates that there was no evidence of a difference in the reduction in weekly headache activity amongst the three groups (χ^2_{kw}=5.69, p=0.06).
- Advantages:
  - Simple
  - Unequal samples
  - Popular
  - Powerful

When To Use Which Test

		RESPONSE		
NO OF SAMPLES		NOMINAL	ORDINAL OR NON-NORMAL	NORMALLY DISTRIBUTED
ONE SAMPLE		χ^2-test, Z-test	Kolmogorov-Smirnov Sign test	t-test
TWO SAMPLE	INDEPENDENT	χ^2-test (r x c), Fisher's exact test	Mann-Whitney U Median test	Unpaired t-test
	PAIRED	McNemar's test Stuart-Maxwell test	Wilcoxon signed rank Sign test	Paired t-test
MULTIPLE SAMPLES (K>2)	INDEPENDENT	χ^2-test (r x k) Fisher-Freeman-Halton	Kruskal-Wallis test Median Test Jonckheere-Terpstra test	Analysis of variance (ANOVA)
	PAIRED	Cochran Q test	Friedman test Page test Quade test	Repeated measures ANOVA
ASSOCIATION BETWEEN TWO VARIABLES		Contingency coefficient Phi, r_ϕ Cramér, C	Spearman's rank Kendall's tau	Pearson product moment correlation
AGREEMENT BETWEEN TWO VARIABLES		Simple kappa	Weighted kappa	Limits of agreement

Jonckheere-Terpstra Test

- Test for ordered alternatives or non-parametric test for trend
- Used when groups have explicit order
- More powerful than KW for ordered groups
- Hypotheses:
  - H_0: No difference in medians across groups
  - H_A: Medians increase in predetermined order

Assumptions

1. Data are randomly selected set of observations
2. Data are continuous
3. Groups are ordered in predetermined order
4. Each sample is from the same population

Method

- Construct hypotheses and decide on α
- Specify order of groups
- Tabulate data with groups in order
- For each group order data smallest to largest
- Count the number of times an observation in the first group is lower than ones in others
- Add half to each count for ties
- Do same for other groups and sum to get *J*
- Compare to values in tables for *J*

Example

- Mcm-2 collected in breast cancer study
- Median Mcm-2 expected to increase with grade

Histological grade		
1	2	3
1.99	4.40	6.94
3.01	9.82	8.04
4.17	10.23	9.82
7.13	11.99	15.75
9.82	11.99	18.30
9.91	13.17	25.01
	13.20	26.40
		28.17

Example

1. H_0: Median Mcm-2 same across groups
   H_A: Median Mcm-2 increases with grade
   α = 0.05
2. Groups ordered by grade
3. Create table
4. Order within groups

Example

Histological grade		
1	2	3
1.99	4.40	6.94
3.01	9.82	8.04
4.17	10.23	9.82
7.13	11.99	15.75
9.82	11.99	18.30
9.91	13.17	25.01
	13.20	26.40
		28.17

Precedent counts		
Grade 1 & 2	Grade 1 & 3	Grade 2 & 3
7	8	8
7	8	5 + ½
7	8	5
6	7	5
5 + ½	5 + ½	5
5	5	5
		5
Total: 37.5	41.5	38.5

- J = 37.5 + 41.5 + 38.5 = 117.5
- 117.5 > 99 (n_1=6, n_2=7, n_3=8, α = 0.05)

- Presentation of the results:
  - The Jonckheere-Terpstra test show that there is a trend for an increase in median Mcm-2 value as histological grade increases (J=117.5, p=0.003).
- Advantages:
  - Allows for order in groups
  - More powerful when groups are ordered
  - No need for post-hoc tests
- Limitation: Order pre-specified

When To Use Which Test

		RESPONSE		
NO OF SAMPLES		NOMINAL	ORDINAL OR NON-NORMAL	NORMALLY DISTRIBUTED
ONE SAMPLE		χ^2-test, Z-test	Kolmogorov-Smirnov Sign test	t-test
TWO SAMPLE	INDEPENDENT	χ^2-test (r x c), Fisher's exact test	Mann-Whitney U Median test	Unpaired t-test
	PAIRED	McNemar's test Stuart-Maxwell test	Wilcoxon signed rank Sign test	Paired t-test
MULTIPLE SAMPLES (K>2)	INDEPENDENT	χ^2-test (r x k) Fisher-Freeman-Halton	Kruskal-Wallis test Median Test Jonckheere-Terpstra test	Analysis of variance (ANOVA)
	PAIRED	Cochran Q test	Friedman test Page test Quade test	Repeated measures ANOVA
ASSOCIATION BETWEEN TWO VARIABLES		Contingency coefficient Phi, r_ϕ Cramér, C	Spearman's rank Kendall's tau	Pearson product moment correlation
AGREEMENT BETWEEN TWO VARIABLES		Simple kappa	Weighted kappa	Limits of agreement

Friedman Test

- Extends Wilcoxon-Signed Rank Test
- Tests whether groups have same distribution
  - Examines ranks at different times/matched pairs
- Nonparametric equivalent of repeated measures ANOVA
- Hypotheses:
  - H_0: No difference in medians between groups
  - H_A: At least one difference in medians

Assumptions

1. Data are continuous
2. Data from randomly selected samples
   - Single subject across multiple times/conditions
   - Blocks of matched subjects randomly assigned
3. The subjects/blocks are independent
   - One block or subject has no effect on others

Method

1. Construct hypotheses and select α
2. Construct two way table with N rows, k cols
3. Rank each row from lowest to highest
4. Sum these ranks in each column
5. If null does not hold sum will vary

Method

$$F_r = \frac{12}{Nk(k+1)}\left[\sum_j R_j^2\right] - \left[3N(k+1)\right]$$

- R_j = sum of ranks for column j
- N = number of subjects
- k = number of periods or conditions

5. Look F_r up in tables of Friedman distribution
6. Reject H_0 if F_r is greater than table value

Example

- Taken from Rubin and Peter's paper
- Does hydralazine relieve high bp in lungs

Person	Before		48 hrs after		6 months after	
	Units	Rank	Units	Rank	Units	Rank
1	22.2	3	5.4	1	10.6	2
2	17.0	3	6.3	2	6.2	1
3	14.1	3	8.5	1	9.3	2
4	17.0	3	10.7	1	12.3	2
Rank sum	-	12	-	5	-	7

$$F_r = \frac{12}{4\times 3(3+1)}\left[12^2 + 5^2 + 7^2\right] - \left[3\times 4(3+1)\right] = 6.5$$

Presentation of Results

- The results of the Friedman test indication there is a difference in median total pulmonary resistance across the three time periods. Therefore, we can conclude that hyralazine alters total pulmonary resistance (p=0.042).

Advantages and Limitations

- Versatile
  - Can be used with randomised block design
  - Multiple observations of a single sample
- Can be used for skewed data
- Medians same, significant difference
- Often called Friedman two-way ANOVA
  - Only looks at within groups not between groups
- Not possible to test for an interaction